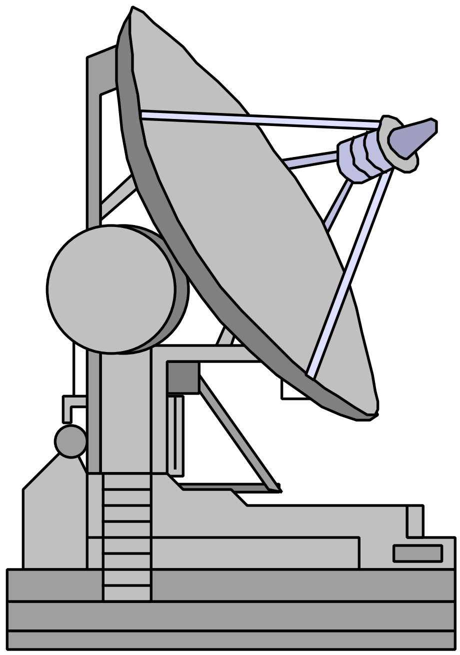


TRACKING DATA PROCESSING

AND

TRAJECTORY DETERMINATION

TABLE OF CONTENTS

INTRODUCTION

This lesson looks at how to track satellites. A question we'll want to answer is 'how do you know exactly where a satellite is and where it is going without ever once touching it?'. In every day life, you do not ordinarily measure the distance to things that you can't touch. For example, to measure the dimensions of a room, the best way is to use a tape measure. With satellites the tape measure approach is no good. Most people take it as a leap of faith that there is *some* way to figure out where a satellite is and where it is going. They might know that it involves ground stations that have big dish antennas and radar, but that's about it. We will discuss the details of the process of <u>Satellite Tracking</u> and <u>Trajectory Determination</u>.

<u>Satellite Tracking</u> is all the steps involved in finding a satellite and taking measurements of its position and velocity. <u>Trajectory Determination</u> is the art of pulling all these measurements together taken from various radar stations and working out the exact path of where it has been as well as figuring out where it will be going next.

To get an understanding of all of these steps, this paper is broken up into three parts. Part I discusses the process that satellite operations employ day to day to track the vehicle and predict where it is going next. Part II is a field exercise. We will be tracking the flight of the balloon, with the intent of bringing the elements of the tracking and determination problem to life. Part III will wrap things up. There we'll review the key points and how satellite and balloon tracking are alike.

Before we get started, there are five main steps involved in the process of satellite tracking and trajectory determination. We will keep coming back to them. They are:

<u>Satellite Tracking</u>

- Finding the spacecraft
- Taking measurements of it’s position and velocity from a given ground station

<u>Trajectory Determination</u>

- Pulling together the measurements taken from different ground stations
- Using equations and all the measurements to predict the orbital path
- How errors in the measurements lead to uncertainty in the orbital path

PART I: PRE FLIGHT DISCUSSION

A. Finding the Spacecraft

Ground stations are not idly waiting for an itinerant spacecraft to wander over head. Instead, they require schedules that tell them a particular spacecraft is coming, when it will get there and what path it should take across the sky. The ground stations need this information to initially find the satellite as well as to drive the dish antenna follow it across the sky. The information about time, place and path where the spacecraft will pass overhead is called Acquisition Vectors. They are based on the last known trajectory of the vehicle. They describe the (expected) trajectory over the ground station.

These Acquisition Vectors don't always work, particularly at launch. At this point the last known trajectory is based on how well you think the launch vehicle did its job, along with the expectation that the spacecraft separated nicely from the final stage of the launch vehicle. If either is wrong, then the spacecraft is just not going to be where expected. For example, people at the ground station will be expectantly waiting for the spacecraft to come over the horizon at 10:52:03 am, at a position of 13 degrees away from due north, but for some reason, the spacecraft doesn't show up.

When acquisition fails, you starts searching for the thing. This is done by radiating commands out to where you think it might be and hoping (frantically) that the spacecraft responds with its own signal. If it does, you can use its signal to pinpoint its position and get back on track. It is exactly like standing in a dark forest calling out for a friend, hoping that they will call back so that you can find them by the sound of their voice. Such satellite searches can be pretty hair raising.

B. Taking Measurements of the Spacecraft Position

Ordinarily, all goes well, and the spacecraft shows up at the expected time and place. During previous contacts, the spacecraft was programmed to turn on its transmitter at the expected time it comes over the horizon of the ground station. The ground stations locks up on this signal. Locking up is rather like tuning into a radio station. The first step is to tune to the frequency of your station. Once you get a garbled babble of voices amidst the static, you can use this to adjust the frequency to make the words understandable. For satellites, first you tune to the expected frequency of the satellite, then you try to find the pulses of frequency variation that represent the bit stream of the digital signal.

The next step is to take measurements of its position and velocity. You might ask: why bother measuring anything if you already know where it is? The answer is that while the acquisition vectors, used to aim the ground antenna, are good enough to pick up the spacecraft's signal, the vectors are not much better than that.

These measurements are used to improve the estimate of the spacecraft's position and velocity. The Acquisition vectors may be mildly incorrect for the spacecraft pass above this ground station. If the estimate is not improved things will be wildly off for the pass over the next ground station. (See more on this topic in the previous section where we discussed frantic searches.)

We primarily use radio frequencies to tracking satellites. We exploit two properties of Radio Frequency (RF) signals to get position and velocity information about the spacecraft. They are (1) Light time delay and (2) The Doppler effect.

1. Light Time Delay used to Determine Range

A burst of RF is radiated from the ground antenna, in the general direction of the spacecraft. Encoded within the RF signal are sequences of bits that tell the spacecraft this signal is meant for it. Each spacecraft has its own unique bit sequence, and will tune out any signals that have a difference bit sequence. If there are other spacecrafts within the path of the signal, they are tuned to ignore this signal. The target spacecraft is tuned to this bit pattern, and when it sees this signal it says 'Aha! A message for me!' It also knows to turn this message around; as soon as it gets the message, it immediately sends one back using the same bit pattern. The ground antenna is waiting for this return message. When it sees the same pattern coming back, it measures the delay time between when it sent the message and when the message came back.

These messages move at the speed of light, which is $300{,}000 \,\frac{\text{kilometers}}{\text{second}}$. For example, if it took 4 seconds to get there and back again then the round trip flight is:

$$300{,}000 \,\frac{\text{kilometers}}{\text{second}} \;\; 4\text{seconds} = 1{,}200{,}000 \text{ kilometers}$$

The one way trip is half of that or 600,000 kilometers.

So altogether, the ground station sends a signal, measures how long it takes for the return reply, divides this by two and multiply this duration by the speed of light. The answer is the distance from the ground station to the spacecraft.

2. Doppler Effect Used to Determine Radial Velocity

When you stand by a road, you can hear cars go by. There are a number of sounds made by passing cars. The first is the motor. You might hear the roar of the engine if the driver happens to gun the engine as the car zooms by. Another sound is that of the tires on the pavement. The tires make that constant background hum you hear as the car passes. If the driver drives at constant speed, and doesn't gun the engine, the tires on the pavement should create a constant sound as well.

But, when you listen, that is not what you hear, regardless of whether the driver is a leadfoot or not. One part of what you hear is the volume sound getting louder as the car bears down on you. A second part of the sound is the frequency of the sound. The whine of the car is at a high pitch as it approaches. Suddenly you'll notice that it changes key just as it shoots by, and the pitch of the sound is much lower as the car takes off down the pike into the distance. This change in pitch of the sound you hear is the Doppler effect. What is happening is that the frequency of the sound waves coming from the car that hit your ear are changed depending on whether the car is moving toward you or away from you.

The same kind of thing applies for radio frequency signals. The shift in frequency of the radio signal is as easy to detect with an antenna as the shift in the car sound is for you to detect with your ears. The antenna has the advantage over your ears that it can measure exactly how much the frequency has changed. These measurements are plugged into equations that convert the frequency shift into velocity of the spacecraft toward or away from the ground antenna. With your ears you must be content with just noticing the frequency change. You can't make an exact measure of the change in pitch.

Doppler Effect and Light Time Delay use tricks with radio frequency signals to get tracking information. In addition to this, you can use the physical orientation of the ground station antenna to get more tracking information. The direction that the antenna is pointing helps pinpoint what part of the orbit the spacecraft is in. We have two pieces of information: the elevation angle, and the azimuthal angle. The elevation angle is the angle that the spacecraft is from the horizon. The azimuthal angle is how far from due north the antenna is facing.

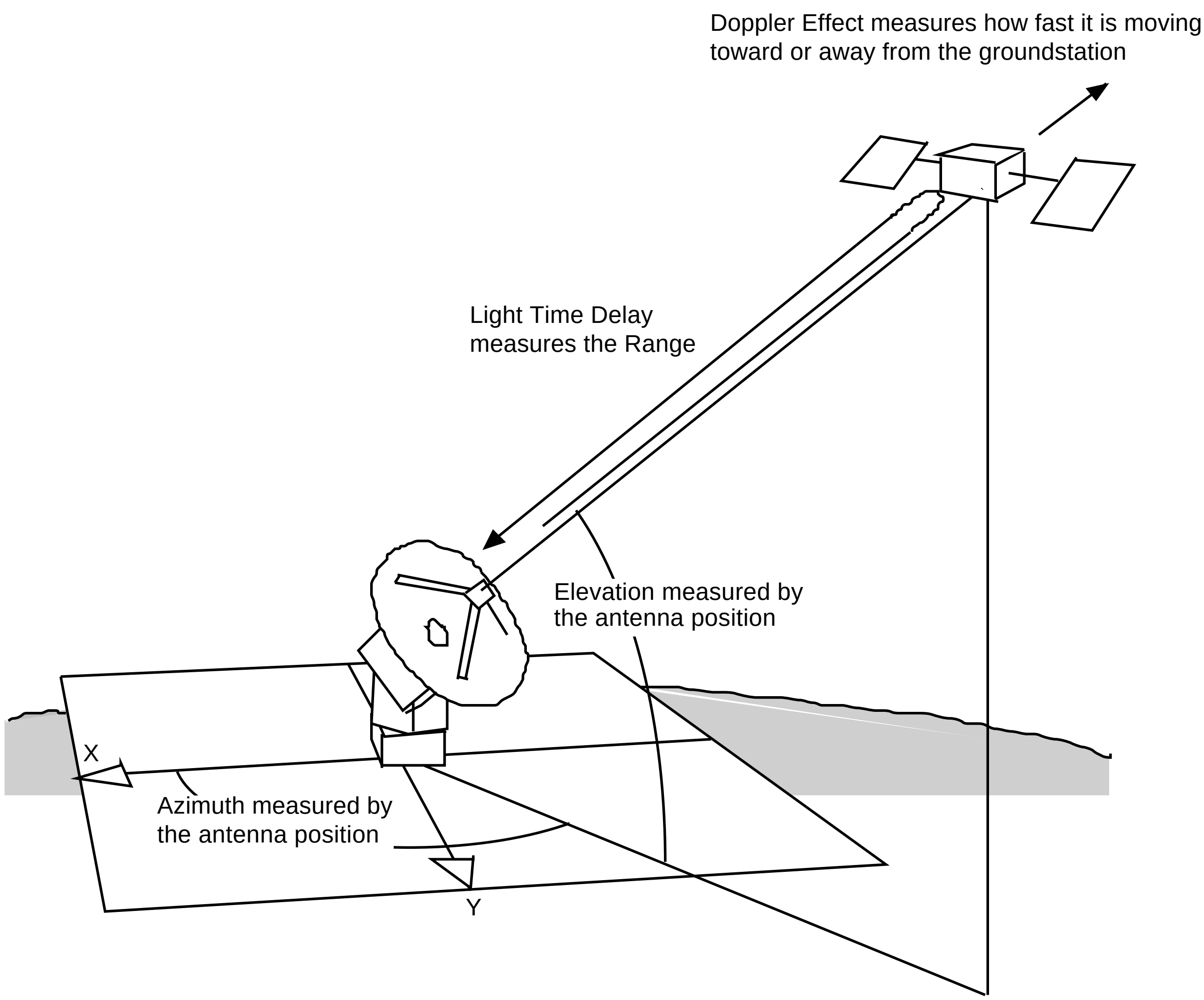


Figure B-1: The Ground station Measures Four Pieces Of Information

C. <u>Pulling Together Measurements Taken From Different Ground stations</u>

We take many measurements in the course of a pass over a single ground station. The best way to compute the trajectory of the spacecraft is to take tracking data from several ground stations.

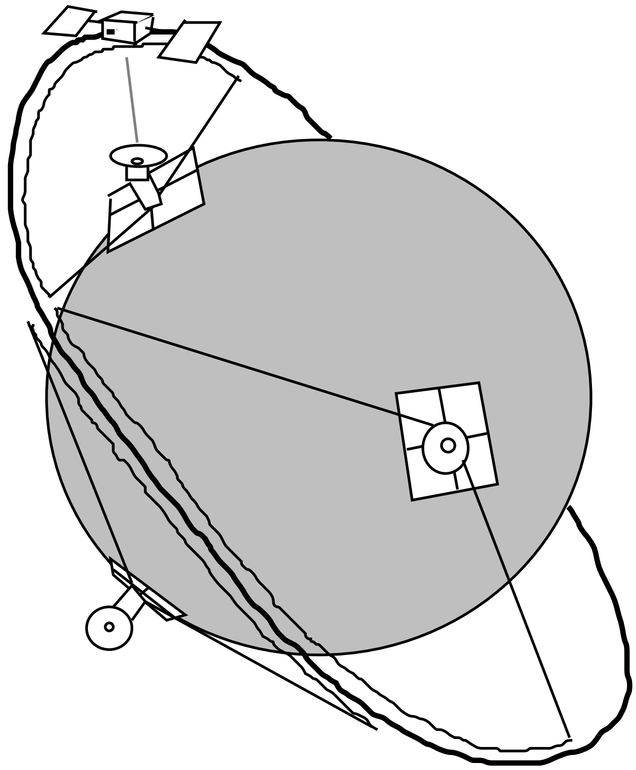


Figure C-1 : Combining Tracking Measurements From Several Ground stations

After the spacecraft has been up in orbit for a while, we take tracking data from several of the most recent ground station passes to compute the orbital trajectory.

D. Using Equations And The Measurements To Predict The Orbital Path

Satellites that orbit the Earth move in circles and ellipses. They are not perfect ellipses or circles. This is because of effects like the air drag on the vehicle, variations in the earth gravitational field, and maneuver burns. Most of these effects are pretty well understood. Generally we make a guess of the satellite's path over several ground stations, take tracking data, compare the guess to actual tracking data, and then improve our guess. In shaping this guess, we use equations that account for air drag, gravitational variations and maneuvers done.

We start with an initial guess of where we expect the satellite to be. At the start of a mission we use data collected by tracking the launch vehicle to derive this starting guess. Later in the mission, we use the results of previous tracking work. In general, the guess of the starting position is a reasonably close stab at the actual starting position. Then we plug the initial guess into a set of equations. The equations chug out tables of the best guess of position and the velocity of the spacecraft.

The next step is to compare this with the tracking data. Does the guess hold up to reality or does the guess need adjustment? Perhaps the initial guess (somehow) predicted that the spacecraft was moving at 3 meters per second as it passed over a particular station. If upon inspection of the tracking data, you see that the velocity was really 7 meters per second, this means that the guess of the path needs some adjustment. The starting guess of the position and velocity is tweaked and the process is done again.

This process of Guess - Compare - Adjust Guess goes on again and again, trying to make the comparison between the guess and the tracking data the best it can be. This goes on until it is impossible to tweak the starting guess any further without making the comparison wander further off instead of getting closer.

This process of making a guess of the path that best fits the data is sort of like drawing a line through a scatter of points. If you had a plot of data points that you somehow knew had to fit the equation of a straight line, you would try the best choice of slope and intercept that would go through the middle of the cloud of data points.

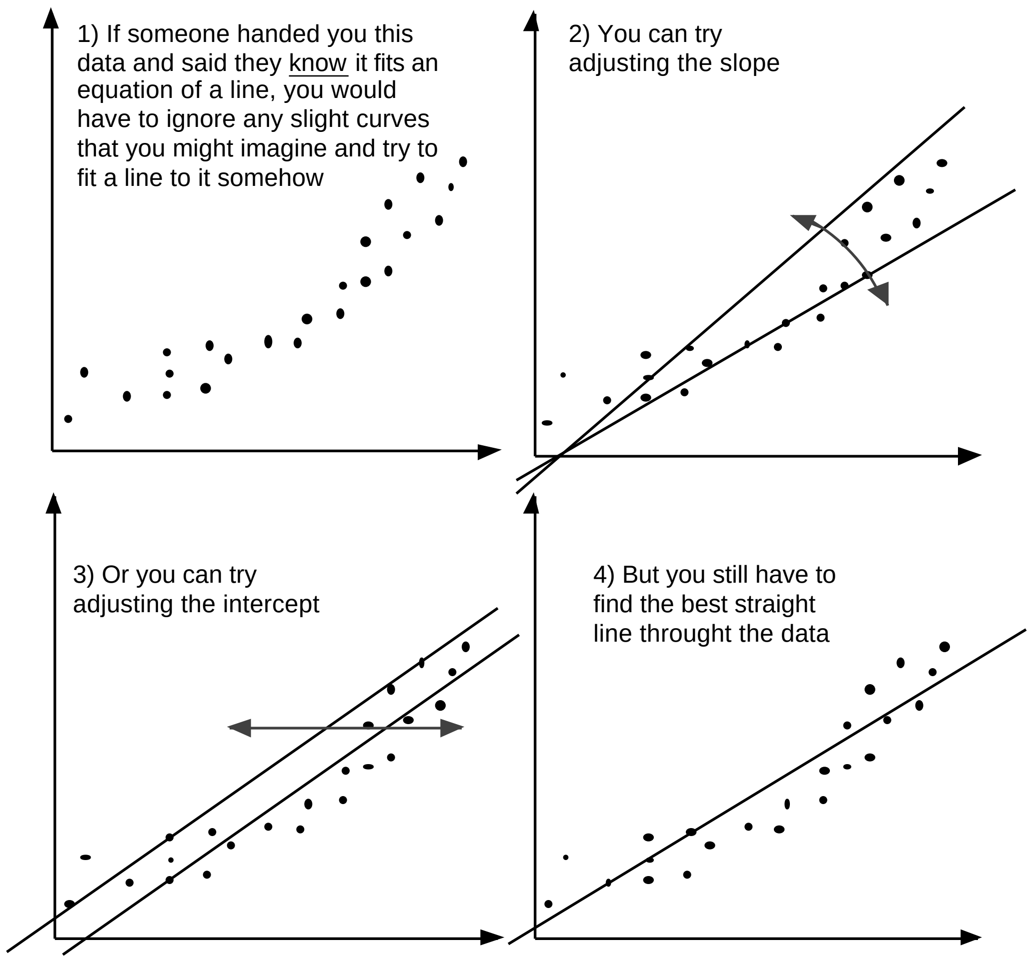


Figure D-1: Adjusting A Line To Fit the Data

If you were given the distribution of points shown in the above figure, you might try to draw a line that seems to go through the center of the cloud of points, you could slide the line up and down or change the tilt a little to try to best get through the center of the cloud. You'd adjust the tilt and height of your line until you squint and say 'well, that's the best I can do.'

In the case of satellite tracking data, the cloud of data points are derived from the range, Doppler, azimuth and elevation information, but since none of the measurements are infinitely accurate, rather than belonging to the equations of a perfect circle or perfect ellipse, we have to pick the best ellipse or circle that goes through the middle of the data.

For orbits, instead of adjusting the equation of a line, we adjust the equations that model the orbit. Instead of changing the tilt and intercept of a line, we adjust the altitude, orbital inclination, air drag, etc., trying to get the orbital path to 'go through the center' of all of the tracking data that you got from all the ground stations. This process of finding the 'best fit' to the tracking data is called Batch Least Squares Estimation. This discussion of 'best fit' is only one of many possible approaches used for determining orbits. Another approach is called a Kalman Filter. Kalman Filters tries to use recent tracking data to always maintain a best estimate of the current position.

The key point you should get out of this discussion is, to compute the trajectory of a spacecraft, you have a set of equations, you plug the tracking measurements into the equations, and out pops an estimate of the trajectory.

E. How Measurement Errors Lead To Uncertainty In The Orbital Path

Measurements are just not perfect. It is no good trying to sweep this under the rug. When you take light time delay measurements, you don't know exactly how long it took to get there and back again. While the accuracy of the measurements are quite good, still in the end it is like being on a long car ride where the driver says "oh, it will take 4 or 5 hours to get there." You might be inclined to ask "Is it 4 hours or is it 5 hours?" There is no way to be certain. In the case of the car ride, the only effect is how long you'll be staring out the car window. In the case of tracking measurements, the uncertainty in the Doppler, the uncertainty in the light time delay and so forth all conspire together to create uncertainty in the satellite's position.

That is why in the last section we were concerned with drawing a curve through the cloud of points. If all the measurements were perfect, there would be no need at all, they would all have fallen along some expected curve and you could have simply connected the dots. In reality however, due to the way the measurements were taken, they don't land where they ought to on the curve, but instead appear somewhere in the vicinity of the appropriate spot.

For example, the light time delay might have been, say, 80.665 milliseconds. The instrument used to measure the delay however might get it wrong and call it 81 milliseconds one time, but the next time around come up with 80 milliseconds, just because that is just the way the electronics work. Were you to plot it you'd get one point above where it ought to be and one point below where it should be. Naturally all the measurements would be equally uncertain. The next thing you know, you've got a cloud of dots that seem to follow some curve, rather than a bunch of dots you can just connect together.

When it comes time to try to guess where the curve is in the cloud, as in the earlier figure, you would be struck with the fact that there are a lot of possible curves, and its just your best guess as to which one to pick. Naturally, you'll try to pick the best one you can, but there will always be an uncertainty, due to all the fuzz. These uncertainties can be quantified, so that for example one might chug though all the equations to conclude that one could estimate the position of a given satellite to within a few kilometers, but you are not sure exactly where it is within those few kilometers.

PART II: FIELD EXCERCISE: Tracking A Balloon's Trajectory

A. Field Exercise Part A

Objective: The point of this exercise is to show you that you can in fact know where an object is without ever touching it. We will also demonstrate some of the difficulties of satellite tracking and trajectory determination. When you are done, you want to have a sense of the main steps involved in the process. These steps are:

1. You take tracking data from a number of ground stations
2. You run the data through some equations
3. You compute the trajectory
4. Your estimate of the trajectory is not going to be completely accurate. There will be errors in you estimate that cannot be swept under the rug.

Rather than work with radio frequency signals for this exercise we'll be using optical wavelengths, primarily because people are equipped with eyes rather than RF antennas. While the equations we'll use as a result of this optical choice will differ from those in RF analysis, the four underlying concepts listed above are the same.

A second purpose of this exercise is to put personal experience behind these concepts. For example, you will not really get a grasp of the significance of the error analysis part of the discussion until you have lived the nightmare with your own data.

Time Requirement: One or two classes of pre-flight discussion
One class for the field experiment
One or two classes of post-flight discussion

Prerequisite skills: Familiarity with the pre-flight discussion, Algebra

Materials: Helium balloon (Bring a couple back-up helium balloons)
One altimeter (See Appendix A)
Two angle measuring tools (See Appendix A)
Two stop watches or timers
A field
Paper and pencil to record data

Procedure: A balloon is floating over a football field. You want to find how far down the field it has floated, how high it has gone and how close it comes to the sidelines. You want to have a continuous track of its path as it floats past. Install two ground stations along the goal line, and measure the distance between them. Station 1 will measure two angles. The first angle is the angle that balloon is above the field. The second angle is measured to the point on the ground directly underneath the balloon. It is the angle that this point makes with the goal line. The second ground station does not need to measure both angles. It only needs to measure the angle to the goal line. These angles are shown in Figure A-1. They have been labeled a, b and c because they will be plugged into an equation presently.
The plan is to call one corner of the football field the origin of a Cartesian coordinate

system, so that the distance the balloon is from the sideline is the X direction, the distance down range is the Y direction and the distance up in the air is the Z direction.

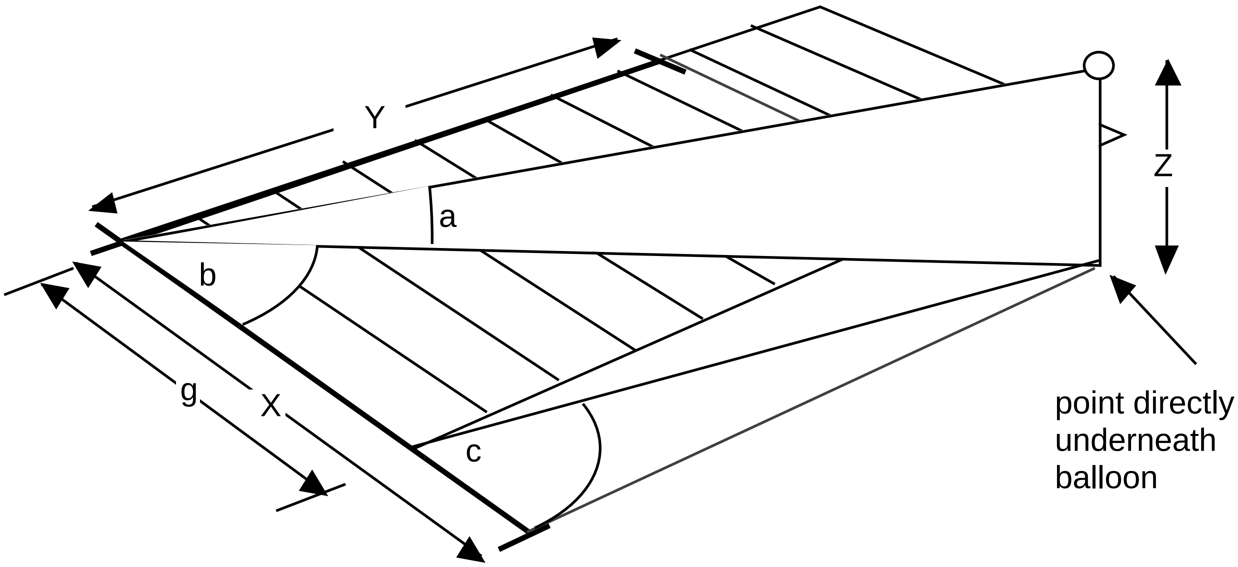


Figure A-1: The Angles a, b and c of the Balloon Position

If you measure the three angles a, b and c, then you can plug them into the following formulas to get X and Y:

$$X = \frac{g \tan c}{\tan c - \tan b} \qquad Y = \frac{g \tan b \tan c}{\tan c - \tan b}$$

Where:

g	=	Distance between the two ground stations, which you measure beforehand with a measuring tape, before looking for balloons
b	=	Angle that the point on the ground directly underneath the balloon makes with station 1. (when it comes time to actually do measuring, this "point underneath the balloon" business will not be any headache)
c	=	Angle that the point on the ground directly underneath the balloon makes with station 2.
X	=	Distance the balloon is from the sideline of the football field.
Y	=	Down range distance of the balloon. It is how far the balloon is from the goal line.

Once you have used angles b and c to determine X and Y, you can use these values of X and Y along with angle a to calculate how high the balloon is in the air using the following formula:

$$Z = \tan a \sqrt{X^2 + Y^2}$$

Where:

a = angle that the balloon makes to the plane of the football field.

Z = height of the balloon above the field.

B. Excercise 1: Simultaneous Measurement

To get you used to measuring things and finding their distance, this exercise asks for the ground stations to make measurements of still objects. Measure the X Y and Z Positions of the tips of the goal posts at the other end of the field. Because they are still the two stations can easily get an accurate fix on their position, as well as simultaneous measurements from both stations.

Using the worksheet, take measurements of a, b and c. Also measure the distance between the two ground stations. Compute the tangents of each and then plug them into the above equations to compute X, Y and Z.

Station 1		Station 2	tan a	tan b	tan c	X	Y	Z
a	b	c						

Figure B-1: Worksheet for Exercise 1

C. Excercise 2: Realtime Tracking

In this exercise, the problem is made more difficult because the balloon will be moving as you try to measure it. It is foolish to try to coordinate the moment of measurement between the two ground stations; they are so far apart, it would be hard to get everyone to measure a b and c all at the same exact instant. Instead, each station will try to take as many measurements as possible, without watching what the other stations are doing. Later, after all the data has been taken, we'll use interpolation to extract coordinated measurements. This problem of uncoordinated measurements between ground stations is actually the same problem encountered with tracking data from real ground stations. We'll adopt some of the same techniques employed there.

1. Finding the Balloon

This particular exercise doesn't illustrate this part of the problem very well. You will know where the balloon is because you launched the balloon, so there is no difficulty acquiring it. If on the other hand somebody were to radio to you "Warning: Balloon coming in your direction from the south-east. Should arrive your position at 10:52 am", then you'd be dealing with the problem of finding the balloon. Balloons are fairly small things when they are up in the sky. You'd want the fellow on the radio to give you a fairly accurate estimate of exactly where and when to look for it, otherwise the balloon might pass by without your finding it.

2. Taking Measurements of the Balloon's Position

Each measurement, a , b and c, will require a team of four people to collect the data. The team goal is to take a lot of measurements quickly an efficiently. The first person, the Gunner, will be in charge of aiming the equipment at the balloon. The Gunner's main task is to keep the equipment targeted on the balloon. A second person, the Recorder, will be in charge of taking the degree reading from the equipment. The third person, the Timekeeper, is in charge of recording the time that each measurement is taken. All of the Timekeepers should synchronize their watches prior to starting this exercise. The last member of the team, the Manager, is responsible for coordinating the aiming and measurement of the team.

As the balloon moves down the field, each team will take as many measurements as they can. Each measurement will go like this:

a. The Manager begins the process by calling "Start Measurement 1".

b. The Gunner will aim the equipment at the balloon. He or she does not have to continuously track the balloon. Instead, the moment the balloon in their sites and they are satisfied that the equipment is well aimed for an instant, they call out "Set" at that moment. It is OK if the Gunner loses track on the balloon after "Set" is called, provided that for that fleeting instant that "Set" was called, the equipment was well aligned on the balloon.

c. The Timekeeper records the exact moment that he or she heard the word "Set" called, along with the measurement number that the Manger declared. The timekeeper also checks off whether they felt it was a good or a bad measurement. For example, if they felt that somehow they missed the moment, or the Gunner says it wasn't a good measurement after all, then the Timekeeper would mark this as a Bad measurement. Don't erase the measurement, just check it as bad. When the Timekeeper is finished writing and is ready to move on to the next measurement, he or she calls out "Done".

d. At the same time, when the Recorder hears the word "Set", he or she writes down the degree marking of the equipment and the measurement number. The Recorder also checks off whether it was a good or a bad measurement. When finished, the recorder also calls out "Done".

e. When the Manager hears both the Recorder and Timekeeper say "Done", and when the Manager feels the team is ready to move on, then the next reading is begun, by calling out "Start Measurement 2"

The forms filled out during flight look like this:

	Flight Num:	1
	Angle Measured	b
Measurement Number	Time	Data Quality
1	10:52:03	Good
2	10:52:09	Good
3	10:52:27	Good
4	10:52:35	Good
5	10:52:55	Bad
6	10:53:12	Good
7	10:53:31	Good
8	10:53:39	Good
9	10:53:53	Good
10	10:54:19	Good

	Flight Num:	1
	Angle Measured	b
Measurement Number	Angle (Degrees)	Data Quality
1	74	Good
2	75	Good
3	72	Good
4	70	Good
5	68	Good
6	56	Good
7	53	Good
8	50	Good
9	53	Good
10	45	Good

Figure C-1: Recording Data for Exercise 2

3. Pulling Together Measurements of the Ground Station

After the flight is over, each team will be plotting results and drawing curves that go through the plotted data. The approach here will be very similar to those employed numerically on computers. The difference will be that the student will be called upon to eyeball a curve through the data collected. This really is not a bad approach. Some might say that computers can fit a line better than an eyeball. This isn't really true. One advantage that computers have is the ability to do it automatically (while the tired user goes off for some coffee). Computers also have the advantage of providing repeatable results. Give the computer the same data twice and it will come up with the same curve fit. Give a human the same data twice and the results will depend on how much coffee has been recently drunk.

a. Plot the Measurement Data

Plot the angle measurements on a graph paper, with the angle measured on the vertical axis and the time of the measurement on the horizontal axis. Don't just plot the Good points from the flight sheets. Plot the Bad ones as well, but circle them so that you can identify them later. There should be 3 plots for each flight, one for each of the angles a, b and c. An example of two of the plots is shown below:

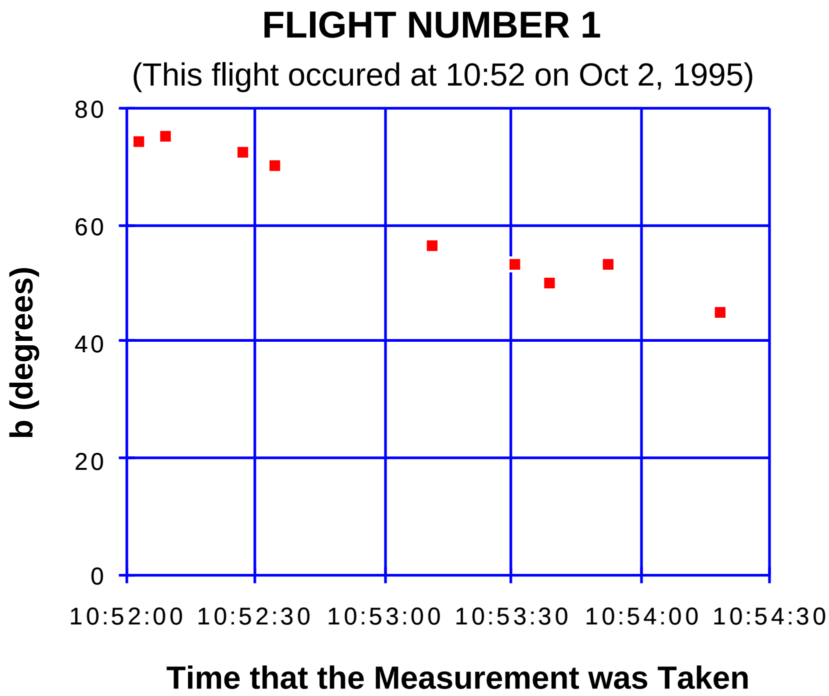


Figure C-2: Sample Plot of Raw b Data

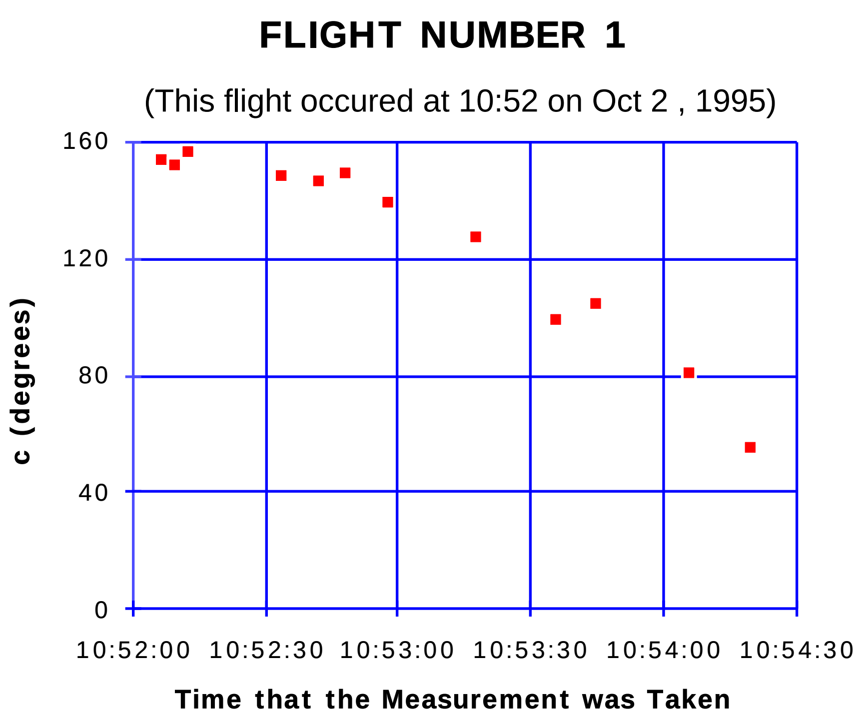


Figure C-3: Sample Plot of Raw c Data

b. Fit A Curve Through the Measured Points

The problem we have with the data from the various stations is that they weren't all taken at the same time. One station might have made measurements every 10 seconds on the dot, another station may have been having trouble so that some measurements came after 5 seconds of aiming, while other measurements took 20 seconds of aiming. The key thing is that in the end, we need to be able to somehow plug in values for a b and c for the same moment in time, even though the measurements weren't all taken at the same moments in time. We will do this by fitting a curve through our data.

The way to do this is to try to draw a curve that seems to go through the middle of the fuzz of dots that make up the plot. If any of your data points are circled as Bad ones, ignore them during the process. A sample of the right way to do it and a couple of wrong ways to do it are shown below:

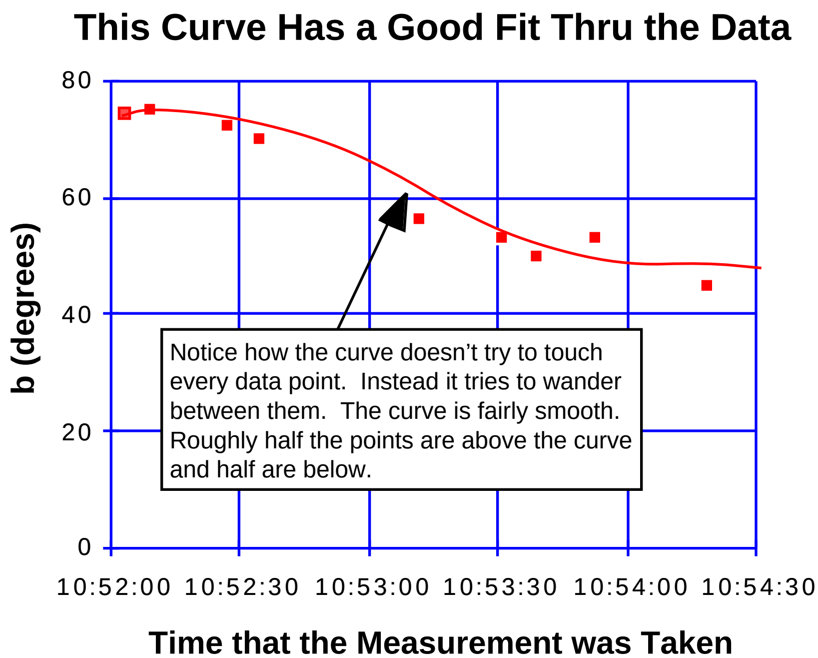


Figure C-4: Good Curve Fit

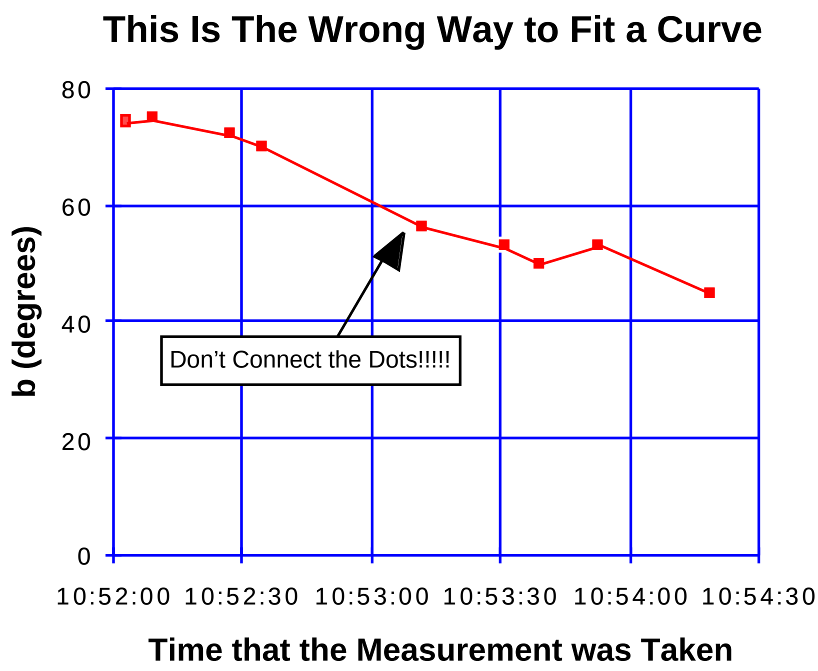


Figure C-5: Bad Curve Fit

c. Using the Curve Fit To Get Coordinated Measurements

The reason we tried to fit the curve through the data points is to allow us to pick off a set of values for angles a, b and c for the moment in time. You can't do this using the original data points, because the teams in each ground station didn't bother coordinating their measurements. For example, we didn't have one person who was in charge of all three stations who shouted "MEASURE!!", that each team took as a cue to write something down. That isn't what real life is like. Uncoordinated measurements are pretty much the way things are done with real tracking stations. Our experiment will suffer the same problems.

From here on out, ignore the original data points and use the curve you drew. The way to proceed is to start on the horizontal axis at some convenient time, say 10:52:30 in our example plots. Then follow straight up until you hit the curve you drew. From there, go straight across to the vertical axis and read the value. If you don't end up right on a tick mark on the y axis, that is OK, just make your best guess as to where it falls between the tick marks. See the example below:

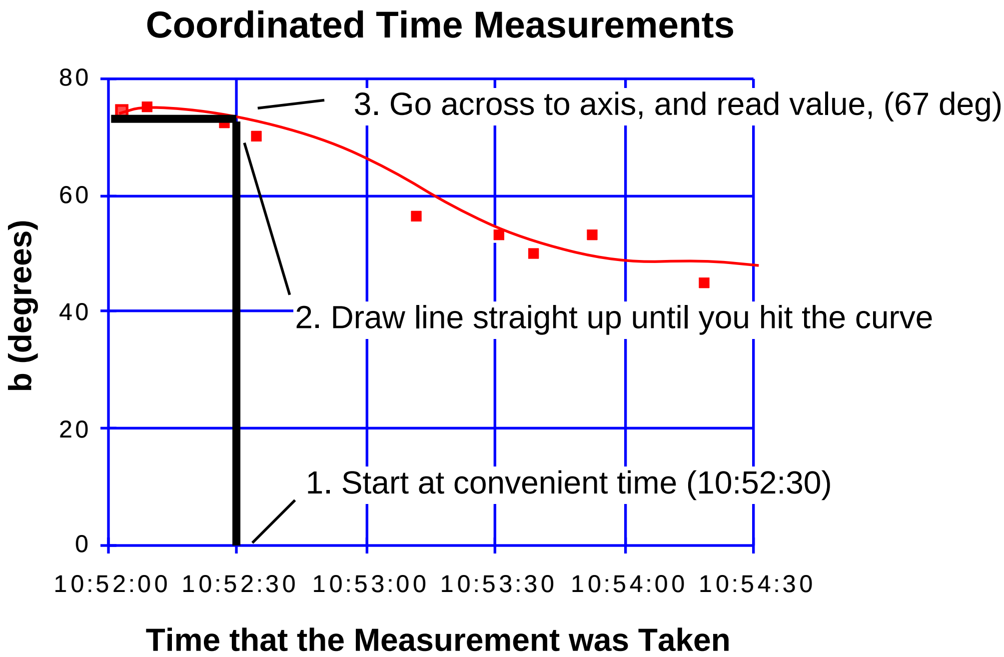


Figure C-6 : Reading a Curve Fit

In the same way read the values for all 3 angles at the common time, in this example, you would have tried to read the values of a, b and c for that one time 10:52:30. Repeat this for a couple of times during the pass. In the example you might have chosen every 30 seconds, say at 10:52:30, 10:53:00 and 10:53:30. Compile a table of times versus interpolated angle:

Time	a (Interpolated)	b (Interpolated)	c (Interpolated)
10:52:30			
10:53:00			
10:53:30			
10:54:00			

Table C-1: Curve Fit Numbers

4. Using Equations and Measurements to Estimate Balloon's Path

The point of the work up to now was to derive *coordinated measurements.* That is to say, the intent was to get values of a, b and c for the same moment in time. Now we can proceed as we did with the first field exercise.

a. Computing X Y and Z From The Interpolated Data

Use the formulae that were shown earlier to compute the values of the down range, side range and altitude of the vehicle during the pass:

$$X = \frac{g \tan c}{\tan c - \tan b} \qquad Y = \frac{g \tan b \tan c}{\tan c - \tan b} \qquad Z = \tan a \sqrt{X^2 + Y^2}$$

Time	a (Interpolated)	b (Interpolated)	c (Interpolated)	X	Y	Z
10:52:30						
10:53:00						
10:53:30						
10:54:00						

Table C-2: Computing X Y and Z

b. Plot the Ground Track and Altitude vs. Time
At this point, you've got the trajectory in the bag. The table of X, Y and Z was the main point of this exercise. From here, you can provide a number of interesting data products. For example, if it was important to know exactly where it was hanging over in each moment, you could generate a ground track of the balloon. To do this, plot X vs. Y on a graph paper, as shown below:

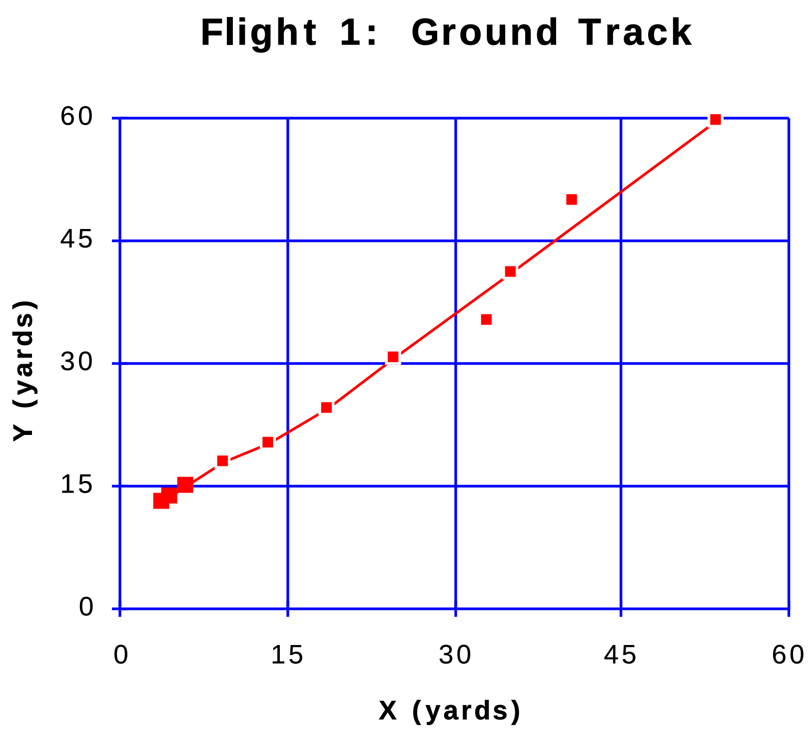


Figure C-7: Ground Track

5. Measurement Uncertainties Become Uncertainty in the Path
You may have noticed that the data points early on in the flight are nice and tight. There is not much choice about where to draw the curve through them. The data points from further away along the balloon path however get kind of wiggly, and drawing a curve between them is a bit of guess work. This is not a coincidence. It was not that the teams were getting tired toward the end of the flight. Instead, the margin of uncertainty was growing.

Imagine that the gunners can find the position of the balloon to within 1 degree. The first diagram below demonstrates that this 1 degree uncertainty is nothing to worry about when the balloon is close by. The way that the example is drawn, the uncertainty in Y is only 1 or 2 yards. On the other hand, 1 degree measurement uncertainty is awful once the balloon gets far away. In the second drawing, when the balloon is down at the other end of the field, the uncertainty in the Y position has grown to 15 or so yards, even though the 1 degree uncertainty is still the same.

When the recorder took the measurement, in our example, he or she just looked at the needle and picked the nearest degree marking, even though the needle was really not quite on either tick mark. Even if the balloon's path was smooth, this process of picking the nearest tick mark means that when you plot the data, it will look like the balloon was zig-zagging back and forth. This is precisely why it is pointless to 'connect the dots' on your plots. None of the dots were exactly right in the first place, they were just the best nearest approximation. When you try to draw the smooth curve, you are trying to guess what the true smooth path of the balloon really was.

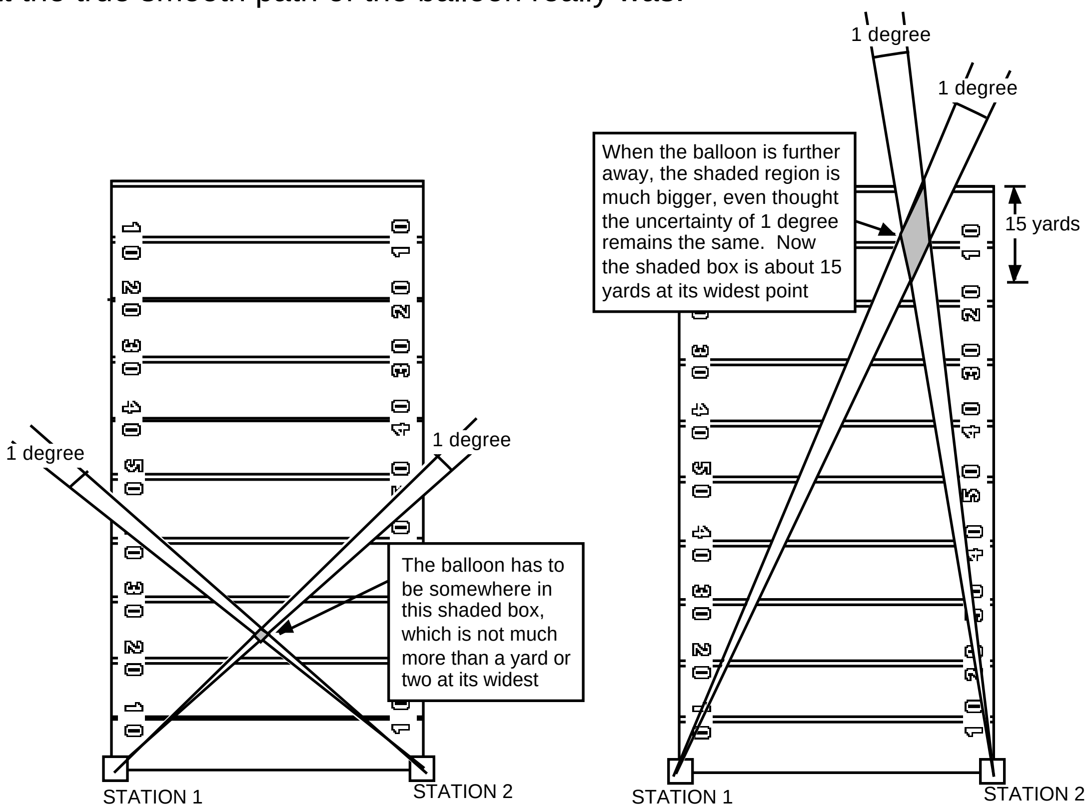


Figure C-8: Error Increases for Distant Positions Even Though Measurement Error Is Unchanged

While fitting the curve from your field exercise, you may have been bothered that there was more than one curve that fit nicely through the data points. This comes about because really, you are just making your best guess of the balloon's path. In reality, it may have followed some other curve through your data. To quantify the error, you can draw another curve that could reasonably go through the data, as shown in the next figure.

Once you have drawn alternative curves, go through the process of computing X, Y and Z all over again using these new curves. If it is too much work to redo the entire thing, just pick a few values to recalculate, perhaps at the end of the curve where the uncertainty is really noticeable. The difference between the answers for X Y and Z for your original curve versus those for your alternate curves is an estimate of the uncertainty in the balloons path through your data.

In the example below, the middle curves for each plot is our best guess curve. Then for each plot was draw alternatives, one which was drawn intentionally on the high side and one drawn on the low side. Now we can for example calculate X three different ways. First would be using the b and c values from middle curve. That is our best guess of X. Then you can try the b and c from the high curve, and finally the b and c for the low curve. To demonstrate, values for low, middle and high are extracted from the graphs for the time 10:53:30. They have been collected into the following table and the values for X in each case have been determined:

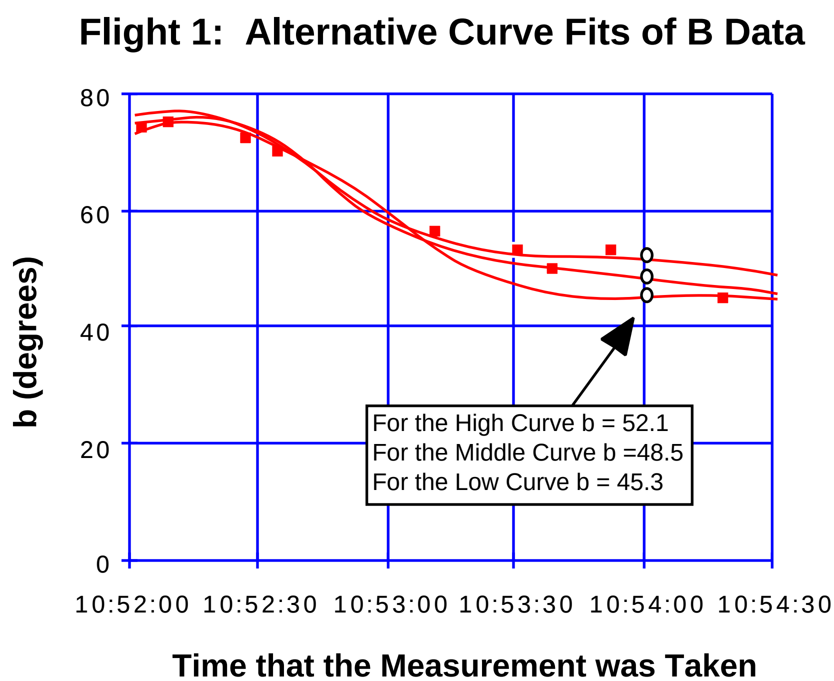


Figure C-9: Alternative Curve Fits of bData

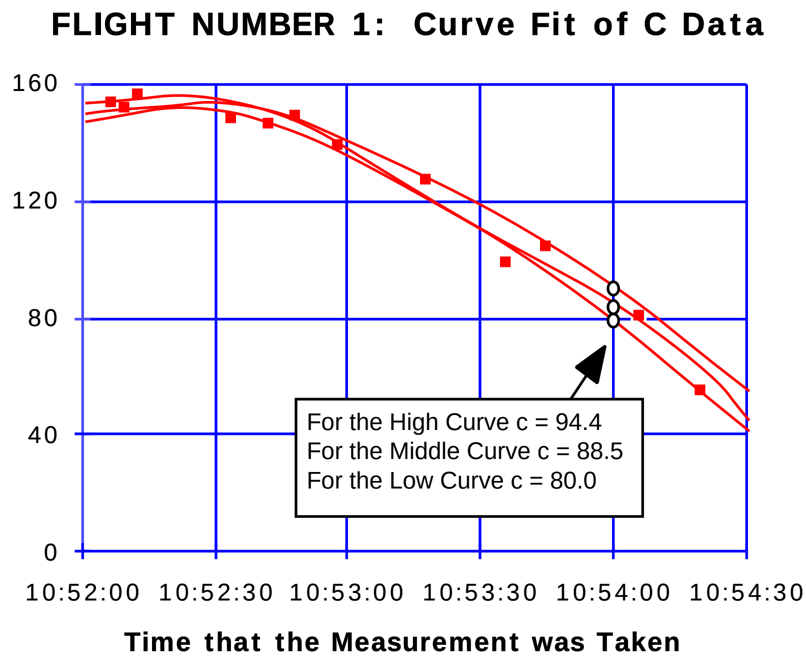


Figure C-10: Alternative Curve Fits of c Data

	b (deg)	c (deg)	X (yards)
High Values	52.1	94.4	
Middle Values	48.5	88.5	
Low Values	45.3	80.0	

g = 30 yards

Table C-3: Quantifying the Amount of Uncertainty

PART III: POST FLIGHT DISCUSSION

This section will go back through the last two parts. First, we'll discuss how the Field Exercise was both like and unlike actual Spacecraft tracking and Orbit Determination. Next we'll reiterate the main points. Then we'll close with a few questions to ponder.

A. <u>How The Exercise Is The Same As Satellite Tracking</u>

1. Common Coordinates

Each ground station makes multiple measurements of range, range rate, azimuth and elevation. The problem is that when the information from the various ground stations is only in local coordinates, those of the ground station. To be able to use them together, we must account for the different positions of the stations around the earth. This is done by converting the data to common coordinates. This common coordinate is one that goes through the center of the Earth. The Z axis of these coordinates goes through the North Pole (true north, not magnetic north). The X axis goes through the equator, at the zero meridian. The zero meridian is the line that goes from north to south, and passes through Greenwich, England. The Y axis of these coordinates is also on the equator, on the 90 degree Meridian. The 90 degree meridian is a line running north to south, it passes a little bit west of Chicago, Illinois.

In our balloon problem, the ground stations are separated by the distance g. The angles a, b and c all by themselves are worthless, unless they are put into common coordinates. In our problem, we set the origin of our coordinates to be underneath station 1. The equations we employed used g, which accounts for the separation between the stations.

2. Error Issue

During the measurement of the balloon in flight, the biggest source of error is trying to decide where the needle is pointing. Is it 56° or is it 57°? The gradations are small, and the needle could as easily be on one as the other. During the experiment you've got to take your best guess and keep on going. If you tried to take the same measurement twice you'd get two answers, regardless of how conscientious you were in taking your readings. The equipment used for satellite tracking suffer the same deficiency. They are good for measuring to within a few number of decimal places. It is pointless to try to pretend they are more accurate than that because the graphs that you could derive from them would have the same kind of problem we saw when we graphed the field data; instead of the points falling along a nice neat curve, they were scattered about, vaguely following some curve that has to be guessed at.

In the field experiment, uncertainty about the angle measurements meant uncertainty about the balloon's position, particularly when the balloon was getting far away. The same exact kind of thing occurs in satellite tracking. Uncertainty in tracking measurements turn into uncertainty about the satellite's position and velocity. These uncertainties are quantified, so that when data is published regarding the spacecraft track, there is also information available as to how many meters of accuracy to expect. Satellite trajectory uncertainty differs from the way we handled the balloon uncertainty. They don't have a team of analysts drawing alternative curves. Instead, there are equations available that can calculate this for them.

B. How The Exercise Is Different From Satellite Tracking

1. Fitting to a Known Equation

Working with wind and balloons is a much more difficult problem than dealing with orbiting satellites. With wind, one is subject to a variety of forces. On a gusty day, the balloons path may zig zag according to the whims of the wind. In a steady breeze the balloon will make straight for the horizon. Predicting the what kind of path the balloon will take tomorrow is as hard as predicting whether it will rain.

This shows up in our exercise because we didn't have any pre conceived notions as to what kind of ground track we were going to get. Straight lines or sharp curves are all possible. Now when the day comes to do the exercise, you can look into the sky and predict what kind of path you'll get. If it is a breezeless day but you come up with wild zig zags, you can guess something went wrong somewhere in your measurements or your calculations. This is because you've got an initial estimate of the path in your head, and you will evaluate the curve you get against this expectation. 'Zig zags won't do on a calm day', you'll say to yourself and draw your curves accordingly.

In the case of satellites things are much more predictable. Satellites going around the earth will be going in either circles or ellipses, with specific modifications allowed to account for air drag and such. Once can employ equations that model all this to draw expected paths through the ground station data. Random zig zags just don't happen.

It actually is easier than our wind problem. What one does with the equations that express the orbit of a satellite is to adjust some parameters such as altitude, orbital inclination and air drag, in the same way as the example above adjusted the slope and the intercept of the line. These orbital parameters are systematically adjusted (using many iterations on a computer) until the best fit can be found. In this case 'best fit' means the choice of orbital parameters that if changed even ever so slightly in any direction only makes matters worse, not better.

2. Didn't Have to Find the Balloon

As has been already pointed out, you didn't have to acquire the balloon. Since you launched it yourself, you had no trouble finding it. Remember however that balloons are fairly small and the sky is awfully big. If you were handed a pair of binoculars and told 'here, find the balloon and track it', you might never find the thing in the first place, let alone tracking it. In that case you would need extra information such as where in the sky to look and when to look there. This is the exact problem faced by ground stations.

C. Summary of the Main Points About Tracking and Trajectory Determination

There is a good deal of information presented here. Some has lasting significance, while other information was just used to illustrate the discussion. The major topics that you should carry away from this are:

1. Finding the Spacecraft

You must know in advance when a spacecraft is going to go over a ground station. You also have to have a fairly good idea of what path it will take overhead. The ground antenna must have both time and path so that it can be in position when the spacecraft goes zooming by.

2. Taking Measurements of it's Position from a given ground station
Ground stations collect information that can be used to figure out the spacecraft's future path. They take many measurements as the spacecraft passes overhead. Each measurement has four things. They are 1) The time delay between when the signal was sent out and when it comes back 2) shift between the frequency of the signal sent out and the frequency of the signal that comes back. 3) Azimuth of the antenna dish at the moment the measurement was taken and 4) Elevation of the antenna disk at the moment of the measurement.

3. Pulling together the measurements taken from different ground stations
To compute the spacecraft trajectory, it is best to use data from various ground stations together. This is done by taking the data measured in the coordinate system of each ground station, and converting it into common coordinates.

4. Using equations and all the measurements to predict the orbital path
Once all the data from the various ground stations have been rendered in common coordinates, it is used in equations that compute the trajectory of the vehicle. We have not gone into much detail about these equations, because it requires differential calculus to properly explain the process.

5. How errors in the measurements lead to uncertainty in the orbital path
It does no good to pretend that you can ignore the error associated with measurements. It is better to recognize the error and try to decide how much uncertainty this will give you about the accuracy of your estimation of the satellite's trajectory.

D. Questions to Test Your Understanding

1. If the ground station antenna can find the spacecraft in the first place, why bother measuring anything? All you are going to get is what you already know. Right?

2. Using data from the last couple of ground station passes you can predict when the spacecraft will go overhead various ground stations in the future. But how does it all get started? At the beginning of the mission, there are not any previous passes to rely on.

3. How can the speed of light be used to find the distance to the spacecraft?

4. Why can't you connect the dots in all the various plots that you have made? Why are you trying to make some vague guess of some curve through the dots?

APPENDIX A: TOOLS FOR MEASURING THE ANGLES a, b AND c

A number of methods can be used. You don't need very fancy equipment however to come up with some satisfying results however.   The homegrown tools shown below have the advantage of being easy to make:

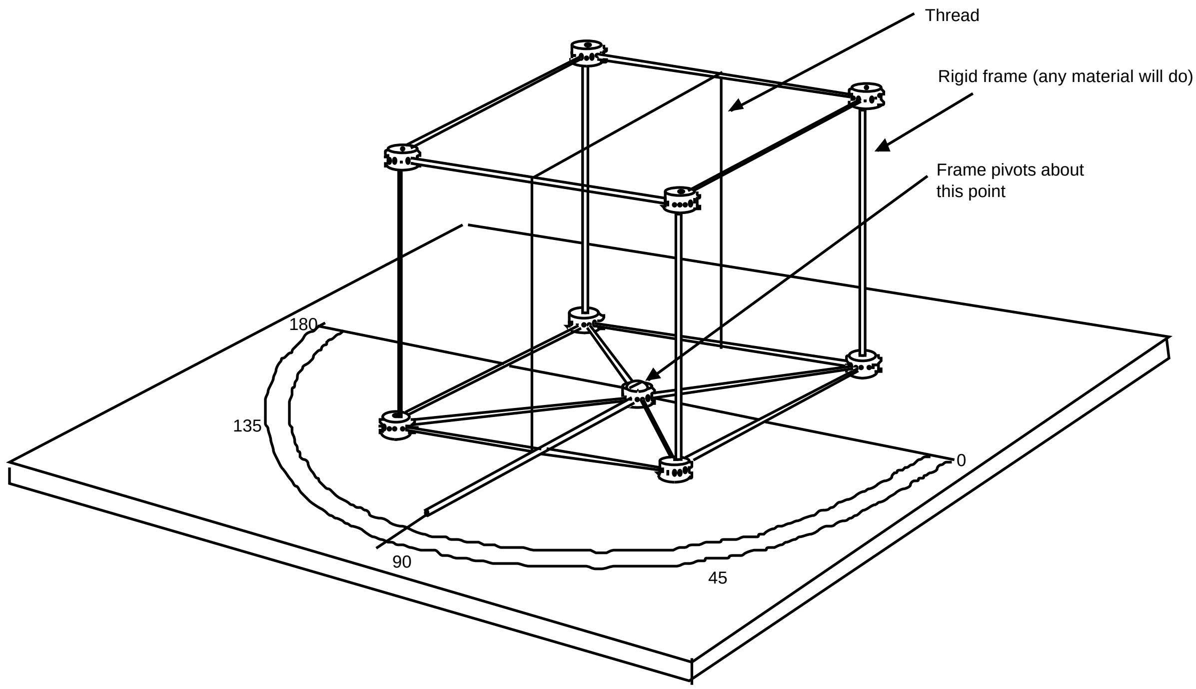


Figure AA-1: Tool For Measuring The Angle To The Goal Line

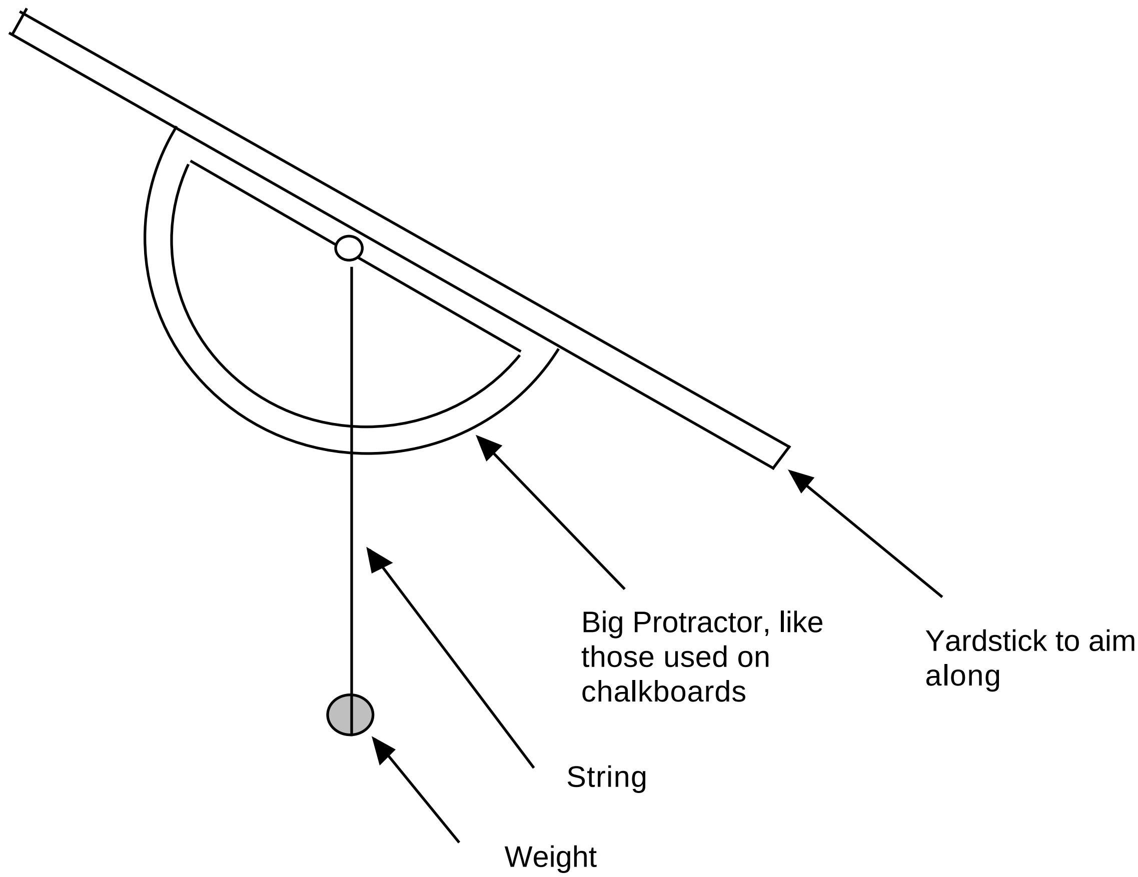


Figure AA-2: Tool For Measuring The Angle Above The Field

APPENDIX B: A COMPLETE SET OF SAMPLE DATA

This section provides a complete set of data and graphs, just in case people get lost in the lesson somewhere. The data employed here is imaginary.

A. FINDING THE BALLOON

As with the Field Exercise, this part of the problem is not addressed.

B. TAKING MEASURMENTS OF THE BALLOON POSITION

The following are the recordings made in the field. Each angle measured has two sheets of paper. One was used by the timekeeper, the other was used by the recorder.

	Flight Num.:	1
	Angle Measured	A
Measurement Number	Time	Data Quality
1	10:52:03	Good
2	10:52:08	Good
3	10:52:12	Good
4	10:52:33	Good
5	10:52:44	Good
6	10:53:12	Good
7	10:53:23	Good
8	10:53:39	Good
9	10:53:45	Good
10	10:54:18	Good

	Flight Num.:	1
	Angle Measured	A
Measurement Number	Angle (Degrees)	Data Quality
1	0	Good
2	2	Good
3	3	Good
4	4	Good
5	5	Good
6	5	Good
7	5	Good
8	6	Good
9	5	Bad
10	6	Good

Flight Num.: 1
Angle Measured: B

Measurement Number	Time	Data Quality
1	10:52:03	Good
2	10:52:09	Good
3	10:52:27	Good
4	10:52:35	Good
5	10:52:55	Bad
6	10:53:12	Good
7	10:53:31	Good
8	10:53:39	Good
9	10:53:53	Good
10	10:54:19	Good

Flight Num.: 1
Angle Measured: B

Measurement Number	Angle (Degrees)	Data Quality
1	74	Good
2	75	Good
3	72	Good
4	70	Good
5	68	Good
6	56	Good
7	53	Good
8	50	Good
9	53	Good
10	45	Good

Flight Num.: 1
Angle Measured: C

Measurement Number	Time	Data Quality
1	10:52:07	Good
2	10:52:10	Good
3	10:52:13	Good
4	10:52:34	Good
5	10:52:42	Good
6	10:52:48	Good
7	10:52:58	Good
8	10:53:18	Good
9	10:53:36	Good
10	10:53:45	Good
11	10:54:06	Good
12	10:54:20	Good

Flight Num.: 1
Angle Measured: C

Measurement Number	Angle (Degrees)	Data Quality
1	154	Good
2	152	Good
3	156	Good
4	148	Good
5	146	Good
6	149	Good
7	139	Good
8	127	Good
9	99	Good
10	104	Good
11	81	Good
12	55	Good

C. PULLING TOGETHER THE GROUNDSTATIONS MEASUREMENTS

1. Make Tables of the Data

Before actually plotting the data, it probably is a good idea to transfer the field data to a table. Notice that if either the timekeeper or recorder marked the data as 'BAD', then a 'BAD' goes in the Data Quality column of the table

FLIGHT: 1

Number	Time	Angle A (degrees)	Data Quality
1	10:52:03	0	Good
2	10:52:08	2	Good
3	10:52:12	3	Good
4	10:52:33	4	Good
5	10:52:44	5	Good
6	10:53:12	5	Good
7	10:53:23	5	Good
8	10:53:39	6	Good
9	10:53:45	5	Bad
10	10:54:18	6	Good

FLIGHT: 1

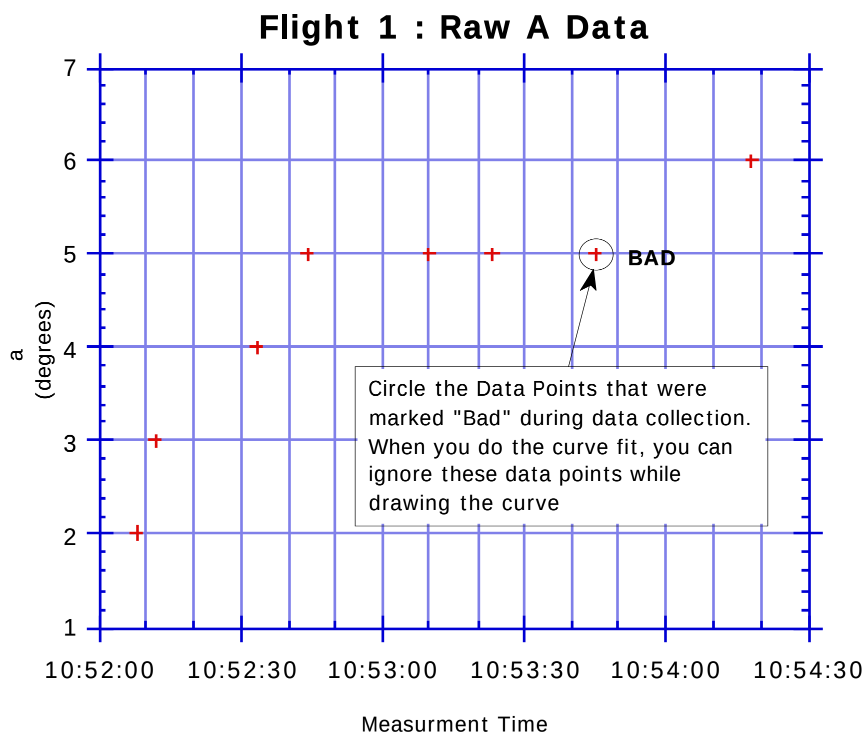
Number	Time	Angle B (degrees)	Data Quality
1	10:52:03	74	Good
2	10:52:09	75	Good
3	10:52:27	72	Good
4	10:52:35	70	Good
5	10:52:55	68	Bad
6	10:53:12	56	Good
7	10:53:31	53	Good
8	10:53:39	50	Good
9	10:53:53	53	Good
10	10:54:19	45	Good

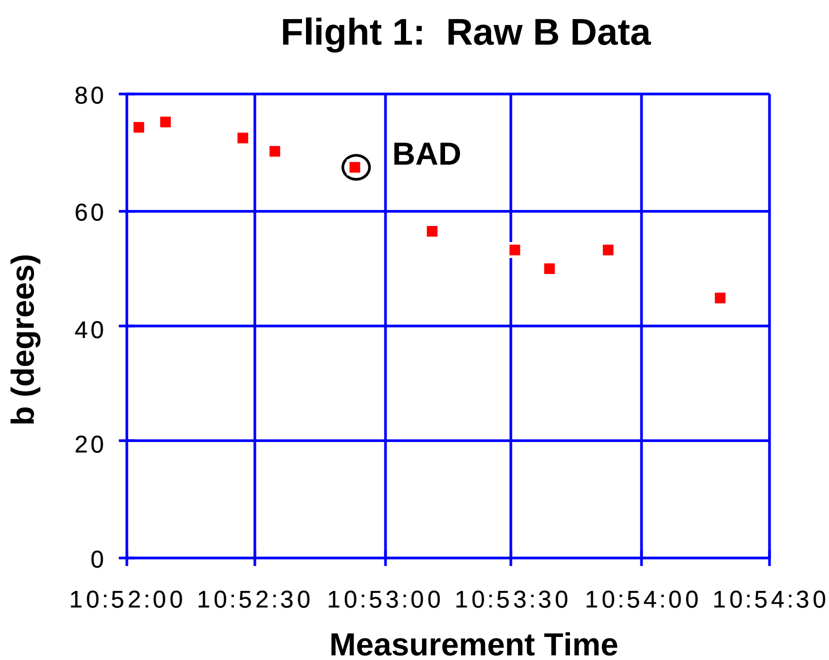
FLIGHT: 1

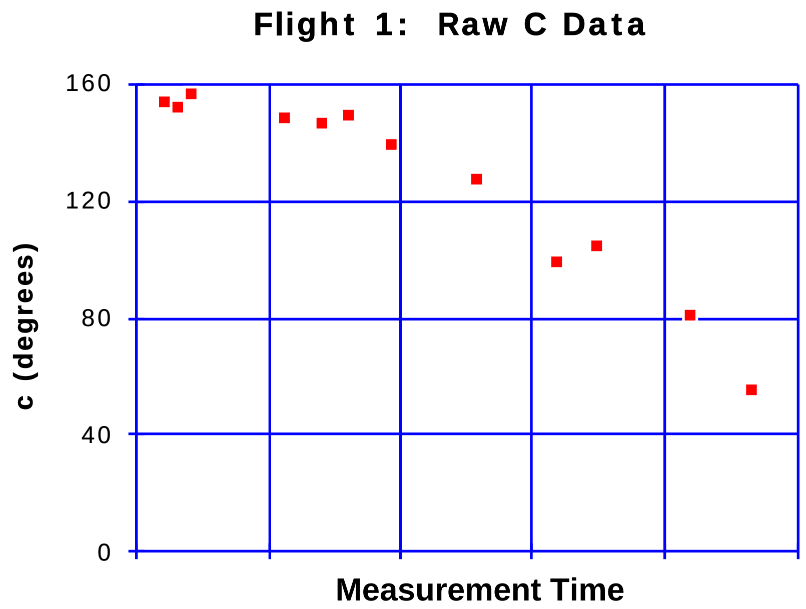
Number	Time	Angle C (degrees)	Data Quality
1	10:52:07	154	Good
2	10:52:10	152	Good
3	10:52:13	156	Good
4	10:52:34	148	Good
5	10:52:42	146	Good
6	10:52:48	149	Good
7	10:52:58	139	Good
8	10:53:18	127	Good
9	10:53:36	99	Good
10	10:53:45	104	Good
11	10:54:06	81	Good
12	10:54:20	55	Good

2. Plot the Data

Now, put the data from the tables into a graph. Any data point that was marked BAD should be circled, so that it can be ignored later when it comes time to curve fit the data.

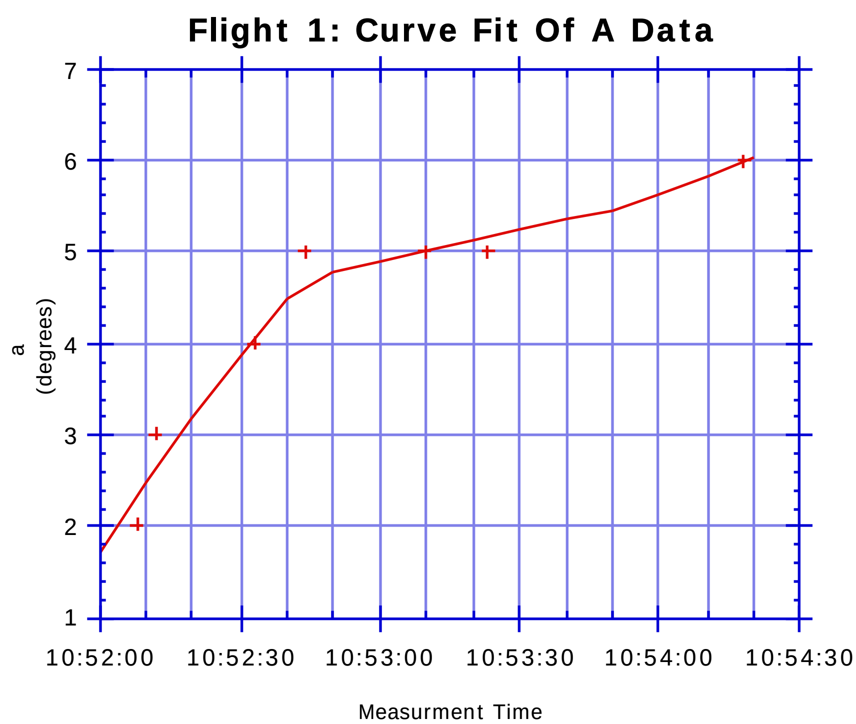




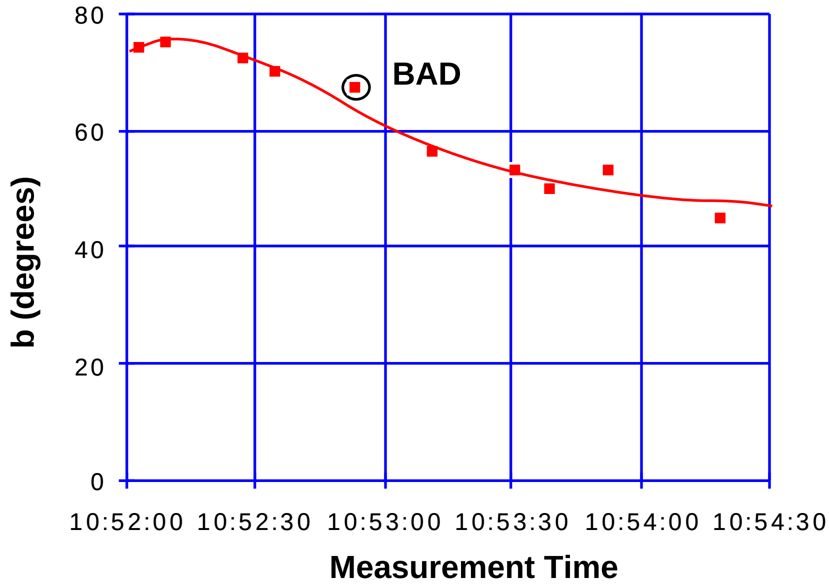


3. Fit A Curve Through the Measured Points

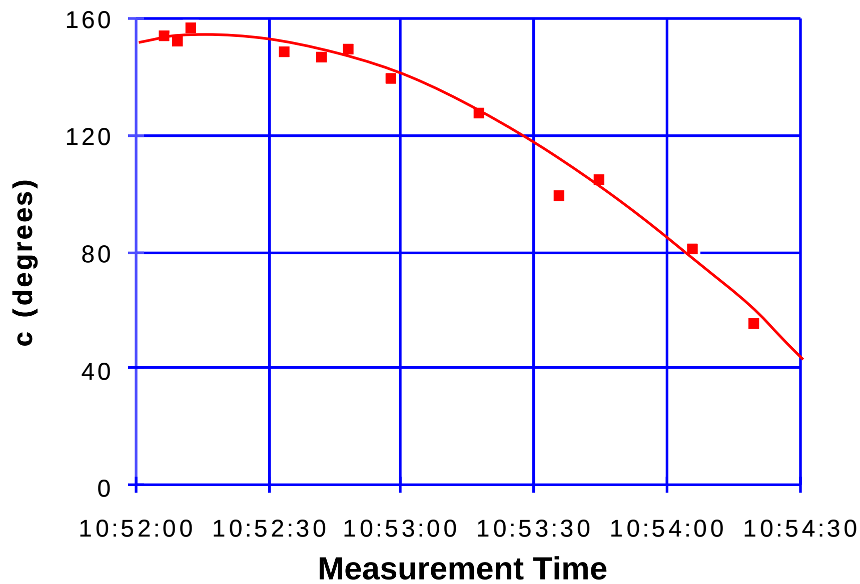
Now it is time to eyeball the best curve through the data points. Don't be alarmed if you find that there is more than one reasonable curve that can be drawn through the points. That is due to the error associated with the measurement.



Flight 1: Curve Fit of B Data



Flight 1: Curve Fit of C Data



4. Using the Curve Fit To Get Coordinated Measurement

Now one must read information off the curve. At this point, forget entirely the original data points you used to draw the curve. Starting at the X axis at a convenient point, for example 10:52:30, go straight up to your curve, and then straight over to the Y axis. In the above curve fit of the C data, you should strike 1.6 on the Y axis.

Flight: 1

Time	a (deg)	b (deg)	c (deg)	x (yards)	y (yards)	z (yards)
10:52:10	2.5	75.7	154.1			
10:52:20	3.2	73.5	153.0			
10:52:30	3.9	71.0	150.8			
10:52:40	4.5	67.7	148.4			
10:52:50	4.8	64.0	145.3			
10:53:00	4.9	60.0	140.9			
10:53:10	5.0	56.1	135.0			
10:53:20	5.1	53.5	128.2			
10:53:30	5.2	51.6	119.1			
10:53:40	5.4	50.6	109.6			
10:53:50	5.5	49.5	97.5			
10:54:00	5.6	49.1	85.5			
10:54:10	5.8	48.5	72.0			

g = 30 yards

D. USING EQUATIONS AND MEASUREMENTS TO SHOW THE PATH

1. Compute X, Y and Z

Now that you have simultaneous values of a, b and c, you can use the equations to fill in the columns for x, y and z. Note that this example assumes that the two stations are 27.4 meters (i.e. 30 yards) apart, so that in the equations, you would use g = 27.4 meters:

$$X = \frac{g \tan c}{\tan c - \tan b} \qquad Y = \frac{g \tan b \tan c}{\tan c - \tan b} \qquad Z = \tan a \sqrt{X^2 + Y^2}$$

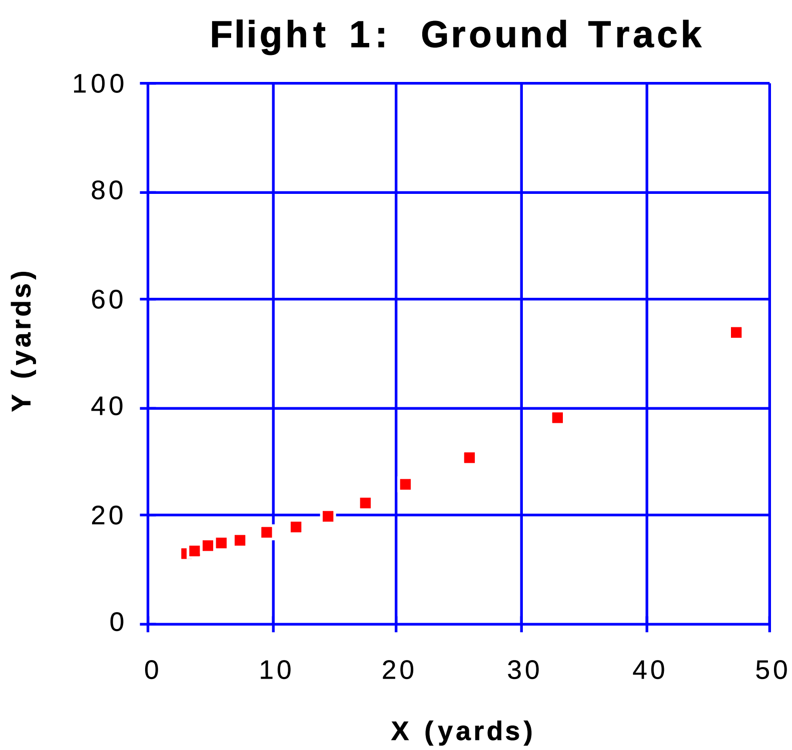
Flight: 1

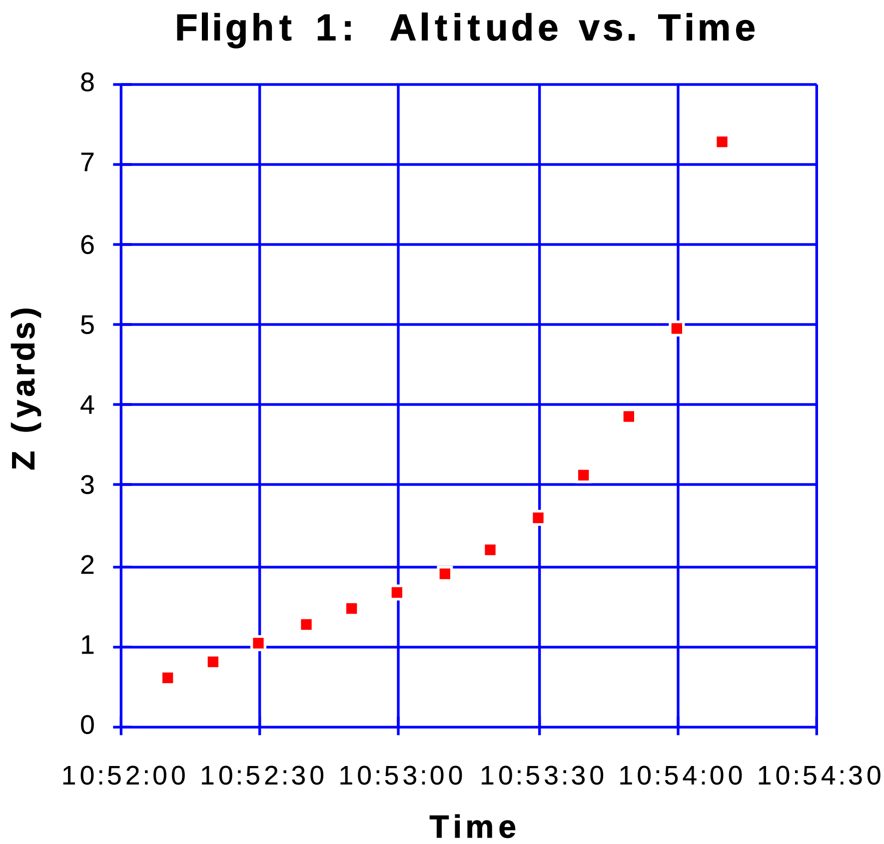
Time	a (deg)	b (deg)	c (deg)	x (yards)	y (yards)	z (yards)
10:52:10	2.5	75.7	154.1	3.30	13.0	.586
10:52:20	3.2	73.5	153.0	3.93	13.3	.775
10:52:30	3.9	71.0	150.8	4.84	14.1	1.02
10:52:40	4.5	67.7	148.4	6.04	14.7	1.25
10:52:50	4.8	64.0	145.3	7.57	15.5	1.45
10:53:00	4.9	60.0	140.9	9.58	16.6	1.64
10:53:10	5.0	56.1	135.0	12.1	17.9	1.89
10:53:20	5.1	53.5	128.2	14.5	19.6	2.18
10:53:30	5.2	51.6	119.1	17.6	22.2	2.58
10:53:40	5.4	50.6	109.6	20.9	25.5	3.12
10:53:50	5.5	49.5	97.5	26.0	30.4	3.85
10:54:00	5.6	49.1	85.5	33.0	38.1	4.94
10:54:10	5.8	48.5	72.0	47.4	53.6	7.27

g = 30 yards

2. Plot the Ground Track and Altitude vs. Time

At this point, the hard work is done, now it is time to derive various interesting plots of the information. In this case, we have plotted Ground Track as well as Altitude vs. Time



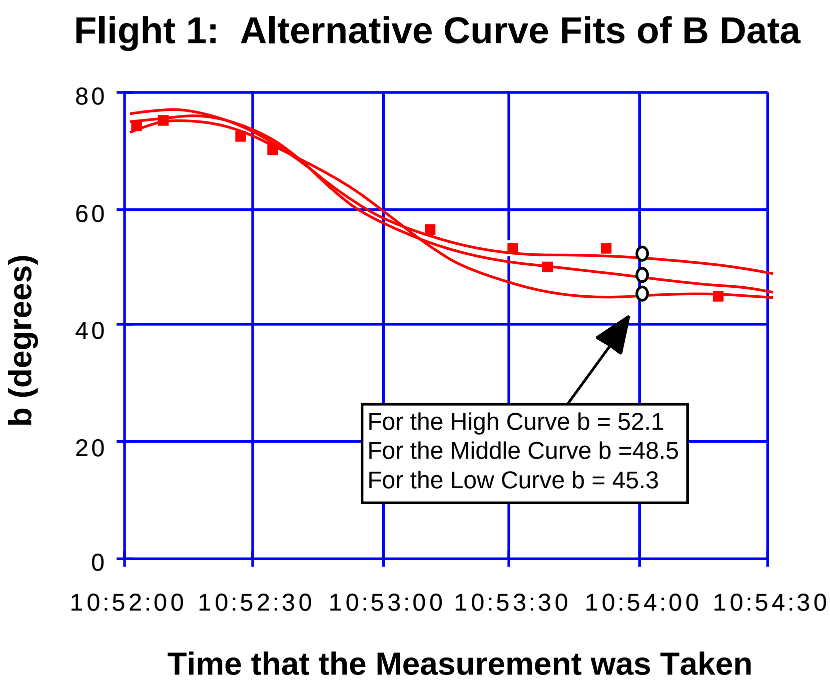


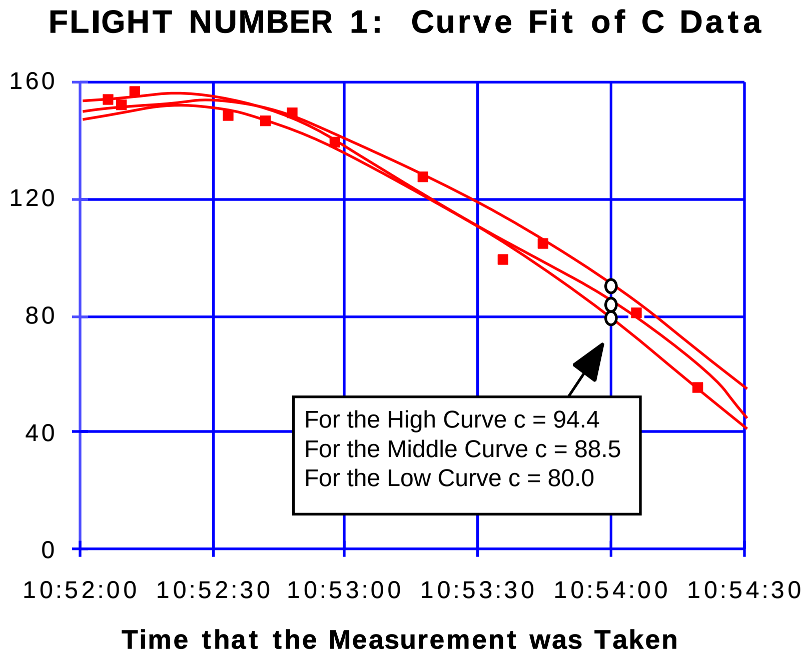
Final Note

Don't get caught up expecting your data to look exactly like this data. For example this data assumes that the wind is going at the same speed all the time. This may not be the case when you collect data. If it is a gusty day, your balloon might not follow a nice straight line. Also, sometimes there is an abrupt change in wind speed just above the tree tops. The balloon may be slow getting started, but suddenly takes off downwind after it clears the trees.

The main thing is that the wind is very hard to predict. Don't expect straight lines and don't expect your data to look like this data.

E. HOW ERRORS IN THE MEASUREMENTS LEAD TO UNCERTAINTY IN THE BALLOON PATH





	b (deg)	c (deg)	X (yards)	Y (yards)
High Values	52.1	94.4	27.3	35.1
Middle Values	48.5	88.5	30.9	34.9
Low Values	45.3	80.0	36.5	36.9

g = 30 yards

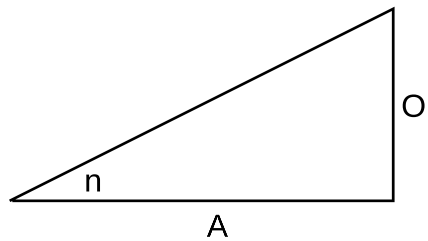
From this table you may estimate that the error in X at this point in the flight is approximately 27 - 37 yards = 10 yards uncertainty. Another way of saying this is that the position of the balloon is 32 yards side range , with an uncertainty of +/- 5 yards.

Similarly the down range uncertainty is 35 - 37 meters = 2 yards. Another way of saying this is that the downrange position of the balloon is 36 yards + / - 1 yards.

A similar set of curve fits and calculations can be done for the altitude, but is not included here.

APPENDIX C: DERVIATION OF THE BALLOON TRACKING EQUATIONS

The key trigonometric equation that we will use is that for the tangent of an angle, shown below:



$$\tan\ n = \frac{O}{A} \qquad \text{(AC-0)}$$

but we are going to have to rotate this triangle to see how we are going to use it:

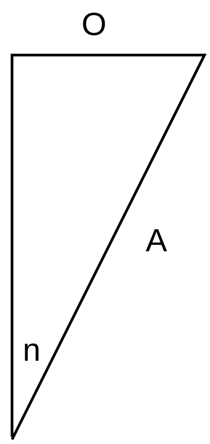


Figure AC-1: Depicting the Tangent Equation on a Triangle

This equation is used twice, once for the angle b and once for the angle c:

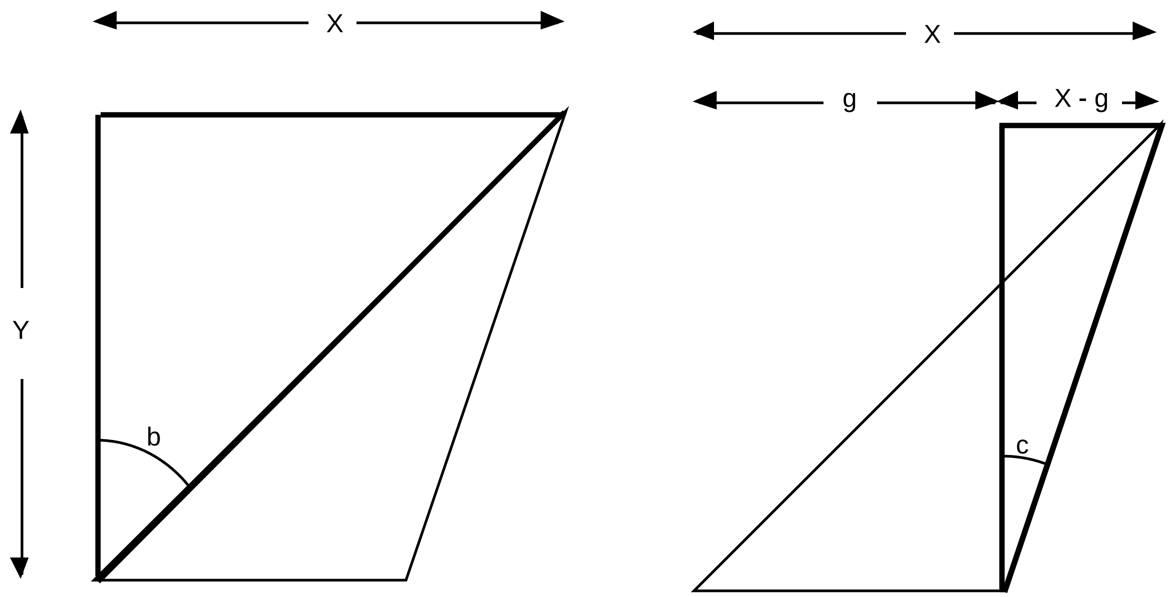


Figure AC-2: X and Y Positions in Triangles

For the angle b, the O in the tangent equation becomes X, while the A becomes Y. Therefore we get that:

$$\boxed{\tan b = \frac{X}{Y}} \tag{AC-1}$$

The angle c shown in the right hand drawing, is slightly more complex, because the top part of the triangle is not just X, instead it is X - g. The A is still Y:

$$\tan c = \frac{X - g}{Y} \tag{AC-2}$$

The first expression for tan b can be solved for Y by first multiplying both sides of the equation by Y:

$$Y \tan b = Y \frac{X}{Y} \tag{AC-3}$$

The Y cancel out on the right hand side, leaving:

$$Y \tan b = X \tag{AC-4}$$

Now both sides can be divided by tan b:

$$\frac{Y \tan b}{\tan b} = \frac{X}{\tan b} \tag{AC-6}$$

They cancel on the left hand side of the equation leaving us:

$$Y = \frac{X}{\tan b} \qquad \text{(AC-7)}$$

Similarly, the expression for tan c can be solve for Y. altogether we end up with two key equations. They are:

$$\boxed{Y = \frac{X}{\tan b}} \quad \text{and} \quad Y = \frac{X - g}{\tan c} \qquad \text{(AC-8, AC-9)}$$

Now since both of the right hand side of these equations equal Y, then you can say that they are equal to each other:

$$\frac{X}{\tan b} = \frac{X - g}{\tan c} \qquad \text{(AC-10)}$$

The plan now is to solve for X. This will take a little bit of algebra:

1) Multiply both sides by tan c :

$$\frac{X}{\tan b} \tan c = \frac{X - g}{\tan c} \tan c \qquad \text{(AC-11)}$$

2) Cancel out the tan c on the right hand side:

$$\frac{X}{\tan b} \tan c = X - g \qquad \text{(AC-12)}$$

3) Do the same thing for tan b: multiply both sides by tan b and cancel. When you do this you get:

$$X \tan c = (X - g) \tan b \qquad \text{(AC-13)}$$

4) Expand the parenthesis on the right hand side:

$$X \tan c = X \tan b - g \tan b \qquad \text{(AC-14)}$$

5) Subtract X tan b from both sides:

$$X \tan c - X \tan b = X \tan b - g \tan b - X \tan b \qquad \text{(AC-15)}$$

6) Cancel out the X tan b on the right hand side:

$$X \tan c - X \tan b = -g \tan b \qquad \text{(AC-16)}$$

7) Collect together the terms in X on the left hand side:

$$X \ (\tan c - \tan b) = -g \tan b \qquad \text{(AC-17)}$$

8) Divide both sides by (tan c - tan b):

$$\frac{X \ (\tan c - \tan b)}{(\tan c - \tan b)} = \frac{-g \tan b}{(\tan c - \tan b)} \qquad \text{(AC-18)}$$

9) Cancel the parenthesis on the left hand side:

$$X = \frac{-g \tan b}{(\tan c - \tan b)} \qquad \text{(AC-19)}$$

10) Multiply the top and bottom of the left hand side by -1:

$$X = \frac{-g \tan b}{(\tan c - \tan b)} \ \frac{-1}{-1} \qquad \text{(AC-20)}$$

11) The negative signs in the numerator go away because $-1 \quad -1 = +1$. The signs of the terms in the denominator are reversed when they are multiplied by the -1 on the outside:

$$X = \frac{g \tan b}{(-\tan c + \tan b)} \qquad \text{(AC-21)}$$

12) Switch the order of the terms in the denominator to make things look neat:

$$\boxed{X = \frac{g \tan b}{\tan b - \tan c}} \qquad \text{(AC-22)}$$

13) We can use one of the first equations to solve for Y as well. Using $\boxed{Y = \frac{X}{\tan b}}$ from above we can say that:

$$Y = \frac{1}{\tan b} X \qquad \text{(AC-23)}$$

14) Now, substitute the equation for X in the box:

$$Y = \frac{1}{\tan b} \frac{g \tan b}{(\tan b - \tan c)} \qquad \text{(AC-25)}$$

15) Cancel out the tan b in the numerator and denominator to give:

$$\boxed{Y = \frac{g}{\tan b - \tan c}} \qquad \text{(AC-27)}$$

The equation for Z is:

$$\boxed{Z = \tan a \sqrt{X^2 + Y^2}} \qquad \text{(AC-28)}$$

Its derivation will be left as an exercise for the reader. Here are some hints:

1) Use the Pythagorean Theorem to derive an equation for L in terms of X and Y
2) Use the equation for the tangent of the angle a to relate Z and L.
3) Solve this equation for Z.
4) Substitute the expression that you get from part 1 of these hints so that you have Z in terms of X, Y and a.

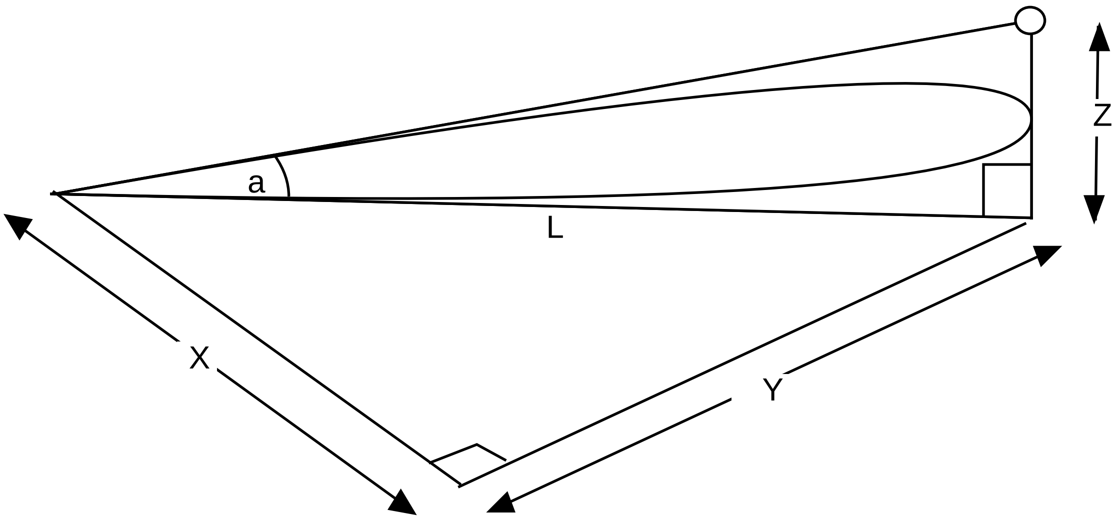


Figure AC-3: Hint for Derivation of the Z Equation